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RESEARCH MEMORANDUM

EXPERIMENTAL INVESTIGATION OF FORCED-CONVECTION HEAT-TRANSFER CHARACTERISTICS OF LEAD-BISMUTH EUTECTIC

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

The forced-convection heat-transfer characteristics of an eutectic mixture of lead and bismuth were experimentally investigated. Data were obtained for lead-bismuth flowing in a circular tube with heat addition and for lead-bismuth flowing in an annulus with heat extraction from the inner surface. Data were also obtained with about 0.04 percent by weight of magnesium added to the lead-bismuth to promote wetting of the heat-transfer surfaces. The investigation covered an over-all range of Peclet numbers from 250 to 3800.

The results of the investigation showed that experimental values of Nusselt number for lead-bismuth fall considerably below values predicted by equations generally used for liquid-metal heat transfer. The addition of magnesium to the pure eutectic, to cause wetting of the heat transfer surfaces, did not change the heat-transfer characteristics of the lead-bismuth.

INTRODUCTION

In view of the potential usefulness of liquid metals in power and propulsion applications where high temperature heat transfer media are required, and because relatively little is known about liquid-metal heat transfer, a research program has been instituted at the NACA Lewis laboratory to obtain fundamental information on heat transfer between liquid metals and surfaces.

This report presents the results of an experimental investigation of the heat-transfer characteristics of lead-bismuth eutectic. Data were obtained for heat addition to lead-bismuth flowing in a circular tube and for lead-bismuth flowing in an annulus with heat extraction from the inner surface. The effect on the heat-transfer characteristics of adding a wetting agent (magnesium) to the lead-bismuth was investigated.

The investigation covered an over-all range of Peclet numbers from 250 to about 3800.

APPARATUS

A photograph of the test setup is shown in figure 1; a schematic diagram of the flow path of the liquid metal is shown in figure 2. 2231 From figure 2, it can be seen that the liquid metal flows into the circulating pump from the storage tank. The liquid metal is pumped through the test and heating sections (shown in more detail in fig. 3), then flows through a cooler and a back-pressure regulating valve into a flow-measuring tank. From the flow-measuring tank, the metal is returned to the storage tank. The components of the setup are described in more detail in the following paragraphs:

Storage tank. - The storage tank serves as a reservoir for the liquid metal when the system is not in operation and maintains a relatively constant head of metal at the pump inlet when the system is in operation. The tank is made of 347 stainless steel and has an inner diameter of 24 inches and a depth of about 13 inches. An air-actuated valve was located at the tank exit to control the filling and emptying of the tank. During operation, the space above the liquid metal was filled with argon.

Circulating pump. - The circulating pump is a Moyno Frame 4C-4, Type SSG, progressing-cavity pump driven by a 5 horsepower electric motor through a v-belt type of variable-speed transmission. The pump rotor and stator were of 316 and 416 stainless steel, respectively. Graphite and asbestos packing was used for the sealing gland. The liquid-metal flow rate was controlled by varying the pump speed.

Test and heating section. - The test and heating section (see fig. 3) consists essentially of one stainless-steel tube suspended within another. The section is mounted vertically and liquid metal from the pump flows upward in the inner tube and downward in the annular passage between the inner and outer tubes. The upper portion of the section is heated by passing an electric current through it. Therefore, in the lower portion, which is the test section proper, the liquid metal is at a higher temperature in the annulus than in the center tube; and heat is transferred from the liquid metal in the annulus to that in the center tube. This arrangement prevents heat from being generated in the liquid metal, which is an electric conductor, in the test section proper. Also, inasmuch as the rest of the circulating system is at the same electric potential as the lower electric contact, no current flows through this part of the system.

The test and heating section was made of 347 stainless steel. The inner tube had an outside diameter of 0.5 inch and a wall thickness of

0.049 inch. The outer tube had an outside diameter of 0.75 inch and a wall thickness of 0.0625 inch. The lower or test section was 40.2 inches long, and the upper or heating section was about 48 inches long. Mixing chambers were provided at each end of the test section for both the inner and outer passages. These chambers contained baffles for mixing the liquid metal prior to measuring the mean temperature. The liquid metal temperatures were measured by means of chromel-alumel thermocouples located downstream of the baffles in the mixing chambers.

Cooler. - The lead-bismuth leaving the test section was cooled to below the maximum permissible pump temperature in a cooler. This was simply a length of stainless steel tubing surrounded by an annular passage through which water was passed.

Back-pressure regulating valve. - An air controlled, 1-inch valve made of 347 stainless steel was used as a back-pressure regulating valve to keep the annular passage in the test and heating section full of liquid metal. This was necessary to prevent burnout of the heating section and to make certain that the test data were taken with the annulus of the test section full of liquid metal. An electric contact located at the top of the heating section was connected with an indicator light to show when the annulus was full.

Flow-measuring tank. - The flow-measuring tank was made of 347 stainless steel. It had an inside diameter of 10 inches and a height of 17 inches. An air-actuated valve located in the bottom of the tank controlled the filling and emptying of the tank. Electric contacts, located at different depths in the tank, were connected to a clock circuit. Operation of the tank was as follows: When a flow measurement was to be made the valve was closed and the tank started to fill with liquid metal. When the liquid metal touched the first contact the clock started. When the tank filled enough for the metal to touch the second contact, the clock stopped and the valve opened. From the tank volume between contacts and the time required to fill this volume, the volume rate of flow was obtained. Thermocouples were located in the tank to measure the temperature and, hence, the density of the liquid metal. The weight flow was obtained from the density and volume-flow rate.

Contacts were provided for two volumes to more easily accommodate high- and low-flow rates. During operation, the space in the tank above the surface of the liquid metal was filled with argon.

Electric equipment. - Power was supplied to the heating section from a 208 volt, 60 cycle supply line through a saturable-core reactor, and a step-down power transformer. The capacity of the heating equipment was 100 KVA with 25 volts at the transformer's secondary terminals. The saturable-core reactor permitted voltage regulation from the maximum of 25 volts down to about 1 volt. The low-voltage leads of the transformer were connected through copper bus bars and flexible leads to the heating section. About 70 KVA of additional electric power was available for the starting heaters. These heaters were of nichrome wire insulated by ceramic beads. They were wrapped around each component of the setup and connecting piping for the purpose of heating the entire system above the melting point of the liquid metal.

Chromel-alumel thermocouples were peened into the walls of the various pieces of apparatus to measure the temperatures of the components. 2231

Miscellaneous. - The various components of the setup were connected by either welded or flanged joints. All welds were made with 347 stainless steel welding rod. All flanges were of 347 stainless steel with Anchor Packing Company, Ankorite 484-A gaskets.

The entire apparatus was thermally insulated by a covering of 85 percent magnesia and insulating cement.

Provision was made for purging the entire system with argon.

PROCEDURE

The test procedure was as follows: The entire system was purged with argon for one hour. Starting heaters were then turned on and the apparatus was heated until all temperatures were above the melting point of the liquid metal. The pump was then started and the storage tank valve opened, beginning operation. The main power supply to the heating section was then turned on and set at the desired level; the starting heaters were turned off. Fluid velocity was set at the desired level by adjusting the pump speed. After equilibrium conditions had been obtained, all the fluid temperature readings were recorded and the flow rate was measured. This procedure was repeated for a range of flow rates.

SYMBOLS

The following symbols are used in this report:

a a constant

b a constant

B a function defined by equation (18a) of the appendix

c_p specific heat of liquid metal in test-section inner passage, Btu/lb, °F

c_p' specific heat of liquid metal in test-section annulus, Btu/lb, °F

D inside diameter of test-section inner passage, 0.0335 ft

D_1 outside diameter of test-section inner passage (inside diameter of test section annulus), 0.0417 ft

D_2 outside diameter of test-section annulus, 0.0521 ft

h film heat-transfer coefficient inside of test-section inner passage, Btu/sec, sq ft, °F

h' film heat-transfer coefficient outside of test-section inner passage, Btu/sec, sq ft, °F

k thermal conductivity of liquid metal in test-section inner passage, Btu/sec, sq ft, °F/ft

k' thermal conductivity of liquid metal in test-section annulus, Btu/sec, sq ft, °F/ft

k_m thermal conductivity of inner tube material (347 stainless steel), Btu/sec, sq ft, °F/ft

L length of test section, 3.35 ft

Nu Nusselt number of liquid metal in test-section inner passage, hD/k

Nu' Nusselt number of liquid metal in test-section annulus, $h'(D_2-D_1)/k'$

Pe Peclet number of liquid metal in test-section inner passage, $\rho VD\ c_p/k$

Pe' Peclet number of liquid metal in test-section annulus, $\rho' V'\ (D_2-D_1)\ c_p'/k'$

Q total heat transferred, Btu/sec

S heat-transfer surface area of test-section inner passage, sq ft

S' heat-transfer surface area of test-section annulus, sq ft

T_i temperature of liquid metal entering test-section inner passage, °F

T_i' temperature of liquid metal entering test-section annulus, °F

T_o temperature of liquid metal leaving test-section inner passage, °F

T_o' temperature of liquid metal leaving test-section annulus, °F

V velocity of liquid metal in test-section inner passage, ft/sec

V' velocity of liquid metal in test-section annulus, ft/sec

W weight flow of liquid metal, lb/sec

X a constant

ΔT average temperature difference between liquid metal in annulus and that in inner passage, $\left[(T_i' + T_o')-(T_o + T_i)\right]/2$, °F

ΔT_f average temperature difference across liquid-metal film on inside of inner passage, °F

$\Delta T_f'$ average temperature difference across liquid-metal film on outside of test-section inner tube, °F

ΔT_m temperature difference across the material of the test-section inner tube, °F

ρ density of liquid metal in test-section inner passage, lb/cu ft

ρ' density of liquid metal in test-section annulus, lb/cu ft

ANALYSIS OF DATA

The heat-transfer coefficients for heat addition to lead-bismuth flowing in the test-section center passage and for lead-bismuth flowing in the test-section annulus with heat extraction from the inner surface are defined as follows:

$$h = \frac{Q}{S\Delta T_f} \tag{1}$$

$$h' = \frac{Q}{S'\Delta T_f'} \tag{2}$$

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The values of Q, S and S' are known since

$$Q = Wc_p \ (T_o - T_i) \tag{3}$$

$$S = \pi DL \tag{4}$$

$$S' = \pi D_1 L \tag{5}$$

The specific heat c_p and all other physical properties of the liquid metal used in this section are evaluated at the arithmetic average of the inlet and outlet temperatures of the pertinent test-section passage (center passage or annulus). The physical properties of the eutectic mixture of lead and bismuth are obtained from reference 1 and are shown as functions of temperature in figure 4. In some cases extrapolation of the data was necessary, and for these conditions the curves are shown dotted.

To complete the determination of h and h' by means of equations (1) and (2), there remains the problem of determining ΔT_f and $\Delta T_f'$. The overall-average temperature difference between the lead-bismuth in the annulus and in the center passage is

$$\Delta T = \frac{\left[(T_i'+T_o') - (T_o+T_i)\right]}{2} \tag{6}$$

The overall-average temperature difference may be divided into three parts

$$\Delta T = \Delta T_f + \Delta T_m + \Delta T_f' \tag{7}$$

The temperature drop through the wall of the inner tube is

$$\Delta T_m = \frac{Q \log_e\left(\frac{D_1}{D}\right)}{2\pi k_m L} \tag{8}$$

where k_m is evaluated at the average of the liquid-metal temperatures in the center passage and annulus. The variation of k_m with temperature is shown in figure 4.

Combining equations (6), (7) and (8) gives

$$\Delta T_f + \Delta T_f' = \frac{T_i'+T_o'}{2} - \frac{T_i+T_o}{2} - \frac{Q \log_e\left(\frac{D_1}{D}\right)}{2\pi k_m L} \tag{9}$$

In order to determine ΔT_f and $\Delta T_f'$, another relationship between ΔT_f and $\Delta T_f'$ must be found. The ratio $\Delta T_f/\Delta T_f'$ can be determined as follows: From equations (1), (2), (4) and (5)

$$\frac{\Delta T_f}{\Delta T_f'} = \frac{D_1 h'}{Dh} \tag{10}$$

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but by definition

$$Nu = \frac{hD}{k} \tag{11}$$

and

$$Nu' = \frac{h'(D_2-D_1)}{k'} \tag{12}$$

Equation (10) may therefore be rewritten, using equations (11) and (12)

$$\frac{\Delta T_f}{\Delta T_f'} = \frac{k'}{k} \frac{D_1}{D_2-D_1} \frac{Nu'}{Nu} \tag{13}$$

Here, in order to analyze the experimental data, an assumption must be made as to the value of the ratio Nu'/Nu. The following relations for Nu and Nu' are suggested in reference 1 and are currently, commonly used for engineering calculations.

$$Nu = 7 + 0.025\ (Pe)^{0.8} \tag{14}$$

$$Nu' = 5.8 + 0.020\ (Pe')^{0.8} \tag{15}$$

It will be assumed in this analysis that the absolute values of Nu and Nu' are not necessarily given by equations (14) and (15), but that the ratio Nu'/Nu, for the purpose of substitution in equation (13), can be obtained from equations (14) and (15). A discussion of the effect of this assumption is found in the appendix.

Accordingly, the ratio, Nu'/Nu, was assumed to be

$$\frac{Nu'}{Nu} = \frac{5.8 + 0.020\ (Pe')^{0.8}}{7 + 0.025\ (Pe)^{0.8}} \tag{16}$$

and equation (13) becomes

$$\frac{\Delta T_f}{\Delta T_f'} = \frac{k'}{k} \frac{D_1}{D_2-D_1} \frac{5.8 + 0.020\ (Pe')^{0.8}}{7 + 0.025\ (Pe)^{0.8}} \tag{17}$$

The Peclet numbers are defined

$$Pe = \frac{\rho V D c_p}{k} \tag{18}$$

and

$$Pe' = \frac{\rho' V' (D_2-D_1) c_p'}{k'} \tag{19}$$

where V and V' are readily determined

$$V = \frac{4W}{\pi D^2 \rho} \tag{20}$$

and

$$V' = \frac{4W}{\pi (D_2^{\ 2}-D_1^{\ 2}) \rho'} \tag{21}$$

The temperature differences ΔT_f and $\Delta T_f'$ can be determined by simultaneous solution of equations (9) and (17).

The heat transfer coefficients h and h' are determined from equations (1) and (2), and the Nusselt numbers Nu and Nu' from equations (11) and (12).

RESULTS AND DISCUSSION

The basic data obtained in this investigation are listed in table I.

Heat transfer with no wetting agent. - Liquid-metal heat-transfer data are generally correlated by plotting Nusselt number against Peclet number. Figure 5 shows such a plot of data obtained in the present investigation with the eutectic mixture of lead and bismuth (44.5 percent by weight of lead). Figure 5(a) shows the first set of runs (numbers 1 to 23), and figure 5(b) shows the second set of runs (numbers 24 to 50). The second set of runs was taken at a later date than the first set; the data of the second set are more consistent due to improved operating techniques. No wetting agent was used in obtaining the data of figure 5, and lack of "tinning" of the surfaces in contact with the liquid metal indicated that no wetting was obtained. Data are shown for heat addition to the liquid metal in the center passage of the test section, and for heat extraction from the liquid metal in the annulus. Included in figure 5 are curves representing equations (14) and (15) (reference 1). 2231

It is seen that the present data falls considerably below the values predicted by equations (14) and (15). The agreement between the present data and the reference curves becomes somewhat better as the Peclet number is increased. As the Peclet number approaches zero, the present data appear to approach a limiting value of Nusselt number which is in substantial agreement with the predicted values for laminar flow. These values are 3.65 for heat transfer at constant wall temperature and 4.36 for constant heat input per unit length of passage (references 2 and 3).

Heat transfer with wetting agent. - Figure 6 shows the data obtained with about 0.04 percent by weight of magnesium added to the lead-bismuth eutectic to promote wetting of the heat-transfer surfaces. Figure 6(a) is the first set of runs (numbers 51 to 75) with magnesium added, and figure 6(b) is the second set of runs (numbers 76 to 100) with magnesium added. The second set of runs was taken at a later date than the first set; both sets were taken at a later date than the runs with no magnesium added. The consistency of the data improves with time due to improvements in operating technique. Included in figure 6 are curves representing equations (14) and (15).

"Tinning" of the surfaces in contact with the lead-bismuth indicated that, whereas the pure eutectic did not wet the stainless steel surfaces, the addition of about 0.04 percent magnesium caused wetting of the stainless steel surfaces. It is seen that the data in figure 6 again fall considerably below the values predicted by equations (14) and (15). The agreement between the present data and the reference curves becomes somewhat better as the Peclet number is increased.

Figure 7 is a summary plot showing the data of figures 5 and 6. Within the accuracy of the experimental data, no noticeable effect of wetting was obtained. A similar result was presented in reference 4, where it was found that the heat transfer between surfaces and mercury was the same, regardless of whether the liquid metal wetted the walls or not.

SUMMARY OF RESULTS

The results of this investigation of forced-convection heat transfer between stainless steel surfaces and the eutectic mixture of lead and bismuth, for Peclet numbers from 250 to 3800, can be summarized as follows:

1. The present experimental values of Nusselt number fall considerably below values predicted by equations which are generally used for liquid-metal heat transfer.

2. The addition of about 0.04 percent by weight of magnesium to the pure eutectic, to cause wetting of the heat-transfer surfaces, did not change the heat-transfer characteristics of the lead-bismuth.

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APPENDIX - ASSUMPTIONS NECESSARY TO ANALYZE THE DATA

Inasmuch as wall temperatures of the inner tube were not measured in the present investigation, only the overall heat-transfer coefficient could be obtained directly from the data. The separate surface coefficients for the inner tube and for the annulus cannot be obtained without introducing assumptions as to their functional relation with other measured parameters such as Reynolds or Peclet numbers.

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Assumption of equation (16). - It was assumed previously that the relations given in reference 1 (equations (14) and (15)) could be used to find the ratio, Nu'/Nu, and that the value of Nu'/Nu thus found applied for the present data. This resulted in equation (16), which is here repeated.

$$\frac{Nu'}{Nu} = \frac{5.8 + 0.020\ (Pe')^{0.8}}{7 + 0.025\ (Pe)^{0.8}} \tag{A1}$$

This may be rewritten, approximately, as

$$\frac{Nu'}{Nu} = 0.83\left[\frac{1 + 0.0035\ (Pe')^{0.8}}{1 + 0.0035\ (Pe)^{0.8}}\right] \tag{A2}$$

The experimental data were previously evaluated using equation (A2), and the results are shown in figures 5, 6, and 7. Using the data of figure 6(b), which has the least scatter, it is found that the data can be represented by the following equations

$$Nu' = 3.28 + 0.0115\ (Pe')^{0.8} \tag{A3}$$

$$Nu = 3.95 + 0.0138\ (Pe)^{0.8} \tag{A4}$$

The data of figure 6(b) and the lines representing equations (A3) and (A4) are plotted in figure 8(a). It is important to note here that equations (A3) and (A4) (and all similar equations mentioned later in the appendix) are not intended as proposed correlating equations for all liquid metals or even for lead-bismuth. These equations are presented to indicate that the data are consistent, that is, it is possible to find equations which will satisfactorily represent the data and also satisfy the requirements of the assumptions. The data presented in this report are insufficient by themselves to justify the proposal of a correlating equation because they are only concerned with one liquid metal and that over a limited range of Peclet number.

Equations (A3) and (A4) have the form of equations (14) and (15) and satisfy the requirements of equation (A2). The constants and coefficients are, however, not in agreement with those of equations (14) and (15). It would therefore be of interest to investigate the effect of changes in the constants and coefficients assumed in equations (A1) and (A2).

Equation (A1) is approximately of the form

$$\frac{Nu'}{Nu} = X\left[\frac{a + b\ (Pe')^{0.8}}{a + b\ (Pe)^{0.8}}\right] \tag{A5}$$

This may be more readily seen in equation (A2), which is exactly of this form with $X = 0.83$ and $b/a = 0.0035$. The effect of changes in the assumed value of X and the assumed values of a and b will be separately considered.

Change in the assumed value of X. - To indicate the effect of a change in X in equation (5a), the data of figure 6(b) is re-evaluated and plotted in figure 8(b) assuming that X equals 1.0 instead of 0.83; that is,

$$\frac{Nu'}{Nu} = \frac{1 + 0.0035\ (Pe')^{0.8}}{1 + 0.0035\ (Pe)^{0.8}} \tag{A6}$$

The equations that best fit the data points of figure 8(b) are

$$Nu = 3.80 + 0.0133\ (Pe)^{0.8} \tag{A7}$$

$$Nu' = 3.80 + 0.0133\ (Pe')^{0.8} \tag{A8}$$

It can be seen from figure 8 that either set of equations (A3, A4) or (A7, A8) fit their respective set of data points about equally well. No conclusion can be drawn from the present data as to the true value of X.

Change in the assumed values of a and b. - It is not necessary to assume the values of both a and b in order to evaluate $\frac{Nu'}{Nu}$ from equation (A5). The value of the ratio b/a is sufficient, and this has been previously assumed equal to 0.0035. It is possible, by using a somewhat different method from that presented in the "Analysis of Data" section, to evaluate the heat-transfer coefficients assuming a value of a, rather than b/a. This is desirable since an estimate of a may be made directly from the previous data (a is the limit of the Nusselt number as the Peclet number approaches zero). This method of calculating the heat-transfer coefficients, which follows, is basically identical to the method presented in the ANALYSIS OF DATA section but the mathematic manipulations are different.

It was assumed that the equations for the tube and the annulus had the same form as equations (14) and (15), respectively. That is, for the tube

$$\frac{hD}{k} = a + b\ (Pe)^{0.8} \tag{A9}$$

and for the annulus

$$\frac{h'(D_2-D_1)}{k'} = X\left[a + b\ (Pe')^{0.8}\right] \tag{A10}$$

Combining equations (A9) and (A10) to eliminate b

$$\frac{hD}{k} - a = \left[\frac{h'(D_2-D_1)}{k'X} - a\right]\left(\frac{Pe}{Pe'}\right)^{0.8} \tag{A11}$$

The heat transferred from the fluid in the annulus to the wall, the heat transferred through the wall, and the heat transferred to the fluid in the inner tube are equal. Thus

$$Q = \pi DLh\Delta T_f = Sh\Delta T_f \tag{A12}$$

$$Q = \pi D_1 Lh'\Delta T_f' = S'h'\Delta T_f' \tag{A13}$$

$$Q = \frac{2\pi L\ k_m\ \Delta T_m}{\log_e \frac{D_1}{D}} \tag{A14}$$

Inasmuch as

$$\Delta T = \Delta T_f + \Delta T_f' + \Delta T_m \tag{A15}$$

there results from equations (A12), (A13), (A14) and (A15)

$$\frac{1}{Sh} + \frac{1}{S'h'} + \frac{\log_e\left(\frac{D_1}{D}\right)}{2\pi\ k_m\ L} = \frac{\Delta T}{Q} \tag{A16}$$

Letting

$$\frac{\Delta T}{Q} - \frac{\log_e\left(\frac{D_1}{D}\right)}{2\pi\ k_m\ L} = B \tag{A17}$$

and combining equations (A11), (A16), and (A17) gives

$$\frac{1}{S\left\{\frac{ak}{D} + \frac{k}{D}\left[\frac{h'(D_2-D_1)}{k'X} - a\right]\left(\frac{Pe}{Pe'}\right)^{0.8}\right\}} + \frac{1}{S'h'} = B \qquad \text{(A18)}$$

Clearing equation (A18) of fractions and arranging the terms in descending powers of h' gives

$$\frac{BSS'k}{Xk'}\left(\frac{Pe}{Pe'}\right)^{0.8}\left(\frac{D_2-D_1}{D}\right)h'^2 + \left\{\frac{BSS'ka}{D}\left[1 - \left(\frac{Pe}{Pe'}\right)^{0.8}\right] - \frac{Sk}{Xk'}\left(\frac{D_2-D_1}{D}\right)\left(\frac{Pe}{Pe'}\right)^{0.8} - S'\right\} h' - \frac{Ska}{D}\left[1 - \left(\frac{Pe}{Pe'}\right)^{0.8}\right] = 0 \qquad \text{(A19)}$$

Equation (A19) is a quadratic in h' which can be solved by the quadratic formula. Equation (A19) includes, in addition to quantities that are known from the geometry of the test setup or from the test data, the two constants a and X. The data of figure 6(b) was recalculated, using equations (A16) and (A19), for values of $a = 3$, 3.5, and 4 and with $X = 0.83$. The three values of a were chosen to bracket the probable values indicated by the data in figure 6(b).

The results for $a = 3$, 3.5, and 4 are shown in figure 9. Included in the figure are equations consistent with the assumptions that best fit the data. These are

for 9(a)

$$Nu = 3.0 + 0.0185\ (Pe)^{0.8} \qquad \text{(A20)}$$

$$Nu' = 2.49 + 0.0154\ (Pe')^{0.8} \qquad \text{(A21)}$$

for 9(b)

$$Nu = 3.5 + 0.016\ (Pe)^{0.8} \qquad \text{(A22)}$$

$$Nu' = 2.91 + 0.0133\ (Pe)^{0.8} \qquad \text{(A23)}$$

for 9(c)

$$Nu = 4.0 + 0.014 \, (Pe)^{0.8} \tag{A24}$$

$$Nu' = 3.2 + 0.0116 \, (Pe')^{0.8} \tag{A25}$$

From figures 8(a) and (9), it can be seen that all four sets of equations ((A3), (A4); (A20), (A21); (A22), (A23); and (A24), (A25)) fit their respective sets of data points about equally well. 2231

<u>Comparison of the various equations</u>. - For the purpose of comparison, the various equations resulting from the different assumptions of the appendix ((A3), (A4); (A7), (A8); (A20), (A21); (A22), (A23); and (A24), (A25)), and the relations given in reference 1 (equations (14) and (15), ANALYSIS OF DATA) are plotted in figure 10. From figure 10, it can be seen that all the equations fall within a relatively narrow band except the relations given in reference 1, which are appreciably higher than the others.

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TABLE I - BASIC EXPERIMENTAL DATA

Run number	W	T_i	T_o	T_i'	T_o'	ΔT	V	V'	ΔT_m	ΔT_f	$\Delta T_f'$	h	h'	Nu	Nu'	Pe	Pe'	Q	$Q/\pi DL$	$Q/\pi D_1 L$
1	4.50	565	600	645	606	44	8.09	9.38	18.3	17.2	8.54	0.907	1.47	17.0	8.35	3377	1182	5.51	15.57	12.54
2	4.97	566	595	635	605	40	8.93	10.36	16.8	15.4	7.81	.926	1.47	17.5	8.39	3747	1312	5.04	14.24	11.47
3	3.40	535	590	646	590	56	6.10	7.09	21.9	23.1	11.0	.801	1.36	15.3	7.80	2600	901	6.54	18.48	14.88
4	2.20	530	627	720	613	88	3.95	4.59	24.5	44.4	19.1	.476	0.892	8.98	4.93	1659	563	7.47	21.10	16.99
5	2.12	535	645	738	620	89	3.81	4.43	26.7	43.6	18.7	.530	0.994	9.91	5.47	1584	540	8.16	23.05	18.56
6	1.92	700	837	936	785	92	3.48	4.05	28.0	45.3	18.7	.575	1.12	9.63	5.61	1285	437	9.21	26.02	20.95
7	3.29	740	795	856	791	56	5.97	6.93	19.4	25.0	11.6	.717	1.24	12.0	6.25	2202	767	6.33	17.88	14.40
8	3.43	775	845	910	835	62	6.23	7.24	25.3	25.2	11.5	.943	1.67	15.3	8.15	2230	774	8.40	23.73	19.11
9	1.97	564	655	723	620	62	3.55	4.12	20.5	29.1	12.4	.810	1.15	11.3	6.36	1458	504	6.27	17.71	14.26
10	2.52	566	639	695	617	54	4.53	5.26	21.1	22.7	10.2	.803	1.44	14.9	8.04	1874	650	6.44	18.19	14.65
11	2.85	570	630	683	617	50	5.12	5.95	19.6	20.8	9.56	.814	1.43	15.1	8.02	2119	739	5.98	16.89	13.60
12	2.73	575	641	699	627	55	4.91	5.71	20.6	23.7	10.7	.753	1.34	13.9	7.44	2020	701	6.31	17.83	14.36
13	3.53	524	570	615	570	45	6.33	7.35	19.1	17.5	8.38	.919	1.55	17.8	9.01	2739	949	5.68	16.05	12.92
14	3.68	576	611	658	618	44	6.61	7.69	14.9	19.7	9.42	.648	1.09	12.1	6.17	2749	962	4.51	12.74	10.26
15	3.90	609	646	689	649	41	7.02	8.16	16.4	16.6	8.01	.861	1.44	15.7	7.96	2847	997	5.05	14.27	11.49
16	2.64	609	676	740	664	60	4.76	5.53	19.9	27.7	12.4	.632	1.14	11.4	6.20	1910	663	6.19	17.49	14.08
17	2.20	606	695	765	671	68	3.97	4.61	22.0	32.1	13.9	.604	1.12	10.9	6.01	1584	546	6.85	19.35	15.58
18	1.77	605	724	811	684	84	3.19	3.71	23.4	42.8	17.8	.487	0.944	8.70	4.98	1263	432	7.37	20.82	16.77
19	2.07	603	711	794	674	77	3.73	4.34	24.9	36.5	15.6	.606	1.14	10.9	6.04	1484	507	7.82	22.09	17.79
20	2.73	607	679	740	663	59	4.92	5.72	22.1	25.4	11.5	.767	1.38	13.9	7.39	1975	686	6.88	19.44	15.65
21	2.49	609	690	756	670	63	4.49	5.22	22.6	28.0	12.4	.714	1.30	12.9	7.00	1793	620	7.06	19.94	16.06
22	2.32	609	695	766	672	67	4.18	4.86	22.3	31.0	13.7	.637	1.16	11.5	6.22	1671	576	6.98	19.72	15.88
23	3.25	618	670	720	665	49	5.86	6.80	19.8	19.9	9.27	.873	1.51	15.8	8.24	2351	820	6.14	17.35	13.97
24	2.75	551	608	665	605	55	4.94	5.74	18.1	25.40	11.5	.6112	1.09	11.59	6.154	2083	719	5.486	15.50	12.48
25	3.23	568	610	660	612	47	5.80	6.75	15.7	21.33	9.97	.6500	1.09	11.89	6.143	2435	845	4.748	13.41	10.80
26	3.89	585	620	659	624	40	6.99	8.12	15.7	16.40	7.90	.8223	1.38	15.30	7.742	2892	1013	4.765	13.46	10.84
27	2.32	550	631	704	616	70	4.17	4.85	21.6	33.58	14.82	.5543	1.01	10.46	5.647	1749	599	6.577	18.58	14.96
28	1.84	553	660	749	630	84	3.49	4.06	23.6	42.56	17.84	.4831	0.928	8.99	5.088	1442	492	7.265	20.52	16.53
29	1.67	552	695	795	640	94	3.01	3.50	26.9	47.67	19.43	.4962	0.981	9.11	5.263	1225	414	8.358	23.61	19.01
30	1.50	554	724	834	646	101	2.70	3.15	28.5	51.76	20.74	.4880	0.981	8.88	5.196	1090	367	8.925	25.21	20.30
31	1.44	552	686	767	622	75	2.59	3.01	21.9	37.80	15.30	.5057	1.006	9.33	5.491	1061	363	6.754	19.08	15.37
32	3.15	561	611	663	611	51	5.66	6.57	18.2	22.35	10.45	.8246	1.203	15.56	6.804	2375	824	5.512	15.57	12.54
33	2.83	575	633	690	628	55	5.09	5.91	18.8	24.84	11.36	.6545	1.153	12.18	6.434	2104	730	5.745	16.23	13.07
34	3.54	602	645	693	649	47	6.37	7.40	17.3	20.17	9.53	.7476	1.275	13.72	7.052	2596	905	5.328	15.05	12.12
35	3.37	610	656	703	659	48	6.07	7.05	17.6	20.64	9.76	.7440	1.268	13.59	6.982	2460	858	5.426	15.33	12.34
36	2.32	605	688	761	672	70	4.18	4.86	21.6	33.69	14.71	.5662	1.045	10.25	5.606	1678	576	6.740	19.04	15.33
37	2.54	605	676	746	670	68	4.58	5.32	20.2	33.14	14.66	.5390	0.982	9.80	5.314	1846	636	6.312	17.83	14.36
38	1.79	579	690	781	661	87	3.23	3.76	22.3	45.69	19.01	.4307	0.834	7.87	4.474	1307	444	6.954	19.65	15.82
39	1.74	579	700	800	666	92	3.14	3.65	23.5	48.45	20.05	.4304	0.838	7.83	4.457	1264	428	7.369	20.82	16.76
40	1.66	578	717	820	666	95	2.99	3.48	25.7	49.21	20.09	.4644	0.917	8.41	4.836	1201	405	8.076	22.81	18.37
41	1.51	576	742	850	673	103	2.72	3.17	27.8	53.84	21.36	.4611	0.936	8.28	4.875	1083	364	8.773	24.78	19.96
42	1.59	575	726	835	667	101	2.87	3.34	26.7	52.96	21.34	.4490	0.898	8.13	4.716	1150	386	8.403	23.74	19.12
43	1.45	545	690	795	637	98	2.61	3.04	23.8	53.09	21.11	.3923	0.795	7.23	4.265	1068	360	7.359	20.79	16.74
44	1.36	541	706	820	636	104	2.45	2.85	25.2	56.62	22.28	.3933	0.804	7.22	4.295	997	338	7.854	22.19	17.87
45	1.32	538	715	832	637	108	2.38	2.77	26.2	58.80	23.00	.3936	0.811	7.22	4.314	968	325	8.177	23.10	18.60
46	1.13	534	759	891	644	122	2.04	2.37	28.2	68.10	25.70	.3698	0.789	6.70	4.109	817	272	8.899	25.14	20.25
47	2.18	550	636	705	615	67	3.92	4.55	21.5	31.75	13.75	.5849	1.088	11.04	6.071	1644	563	6.562	18.54	14.93
48	2.06	555	646	719	622	70	3.70	4.31	21.4	34.06	14.54	.5451	1.029	10.19	5.691	1539	527	6.561	18.53	14.93
49	1.90	555	657	740	628	78	3.41	3.97	22.1	39.33	16.57	.4881	0.933	9.08	5.115	1413	482	6.785	19.16	15.43
50	3.29	587	643	697	645	56	5.92	6.88	21.0	23.81	11.19	.7664	1.314	14.13	7.300	2424	845	6.448	18.22	14.67

TABLE I - BASIC EXPERIMENTAL DATA. CONCLUDED.

Run number	W	T_1	T_o	T_1'	T_o'	ΔT	V	V'	ΔT_m	ΔT_f	$\Delta T_f'$	h	h'	Nu	Nu'	Pe	Pe'	Q	$Q/\pi DL$	$Q/\pi D_1 L$
51	1.853	581	680	762	655	78	3.34	3.88	20.7	40.41	16.91	0.450	0.866	8.22	4.69	1353	464	6.42	18.14	14.61
52	1.765	584	692	782	665	86	3.18	3.70	21.3	45.43	18.75	.416	0.811	7.56	4.35	1285	438	6.67	18.85	15.18
53	1.626	584	706	805	670	92	2.93	3.41	22.1	50.00	20.37	.393	0.777	7.12	4.13	1176	400	6.94	19.61	15.80
54	1.544	584	724	825	675	96	2.78	3.24	24.0	51.38	20.63	.417	0.836	7.51	4.39	1112	375	7.57	21.37	17.21
55	1.342	580	760	875	683	109	2.42	2.82	26.7	59.24	23.08	.404	0.835	7.18	4.33	954	322	8.46	23.89	19.24
56	2.435	601	670	737	662	64	4.38	5.09	19.0	31.22	13.79	.533	0.972	9.70	5.28	1770	612	5.88	16.61	13.38
57	2.389	606	681	750	670	66	4.31	5.00	20.2	32.18	14.17	.552	1.01	9.99	5.46	1728	598	6.27	17.72	14.27
58	2.259	609	690	760	675	68	4.07	4.73	20.5	33.11	14.42	.547	1.01	9.87	5.46	1627	563	6.40	18.09	14.57
59	2.013	609	700	780	681	76	3.63	4.22	20.4	39.05	16.52	.464	0.885	8.37	4.71	1449	495	6.41	18.11	14.58
60	1.860	606	715	799	684	81	3.35	3.90	22.5	41.29	17.21	.486	0.940	8.69	4.96	1328	454	7.10	20.05	16.14
61	2.628	622	686	746	680	59	4.74	5.51	18.8	27.74	12.45	.601	1.08	10.8	5.83	1893	658	5.89	16.63	13.39
62	2.811	630	685	742	682	54	5.07	5.89	17.3	25.58	11.63	.599	1.06	10.7	5.74	2015	703	5.41	15.29	12.31
63	2.959	639	690	744	688	52	5.33	6.20	16.8	23.71	10.95	.630	1.10	11.2	5.93	2103	737	5.28	14.92	12.02
64	2.673	642	700	759	695	56	4.82	5.61	17.2	26.72	12.03	.575	1.03	10.2	5.52	1899	663	5.43	15.33	12.34
65	2.451	641	709	775	698	62	4.42	5.14	18.5	29.83	13.17	.553	1.01	9.80	5.37	1734	603	5.83	16.48	13.27
66	2.234	638	716	786	699	66	4.03	4.69	19.3	32.20	13.96	.536	0.996	9.49	5.25	1581	545	6.10	17.23	13.88
67	1.940	630	734	810	700	73	3.50	4.07	22.3	35.61	15.05	.561	1.07	9.90	5.62	1367	471	7.06	19.95	16.07
68	2.092	631	720	795	696	70	3.77	4.39	20.7	34.55	14.78	.534	1.01	9.45	5.30	1480	510	6.52	18.41	14.83
69	2.025	625	720	796	695	73	3.65	4.25	21.4	36.28	15.37	.525	0.999	9.34	5.27	1439	494	6.73	19.02	15.32
70	1.705	616	731	809	691	76	3.08	3.58	21.8	38.78	15.96	.501	0.980	8.91	5.15	1212	414	6.86	19.39	15.61
71	1.151	585	744	834	660	82	2.08	2.42	20.3	44.88	17.31	.404	0.844	7.18	4.45	818	281	6.40	18.09	14.57
72	1.715	575	658	720	632	60	3.09	3.58	16.2	30.52	12.77	.462	0.890	8.48	4.90	1258	437	4.98	14.07	11.33
73	1.572	564	655	721	620	61	2.82	3.29	16.3	31.64	13.02	.448	0.877	8.30	4.85	1163	402	5.01	14.14	11.39
74	1.538	554	650	719	614	64	2.76	3.22	16.9	33.74	13.85	.434	0.851	8.03	4.71	1138	393	5.17	14.60	11.76
75	1.366	539	655	731	604	70	2.45	2.86	18.1	37.39	14.97	.420	0.845	7.85	4.67	1020	349	5.55	15.67	12.62
76	1.311	565	742	866	666	112	2.36	2.75	25.8	62.1	24.06	.370	0.770	6.67	4.01	944	316	8.12	22.94	18.47
77	1.161	565	781	915	674	121	2.10	2.44	27.6	67.7	25.69	.367	0.779	6.50	3.99	821	275	8.78	24.80	19.97
78	1.109	562	802	942	680	129	2.00	2.34	29.1	72.7	27.21	.363	0.781	6.39	3.95	781	260	9.32	26.33	21.20
79	1.246	565	770	900	674	119	2.25	2.62	28.2	65.6	25.25	.386	0.807	6.83	4.15	882	296	8.94	25.26	20.34
80	1.301	565	755	882	674	118	2.35	2.73	27.4	65.4	25.19	.374	0.783	6.71	4.06	933	312	8.65	24.44	19.68
81	2.051	590	685	761	661	73	3.69	4.30	21.8	35.9	15.27	.538	1.02	9.78	5.48	1490	511	6.82	19.27	15.52
82	1.983	595	696	780	670	79	3.57	4.16	22.4	39.8	16.82	.498	0.950	9.02	5.08	1435	490	7.01	19.80	15.95
83	1.892	598	705	794	676	83	3.41	3.97	22.6	42.6	17.78	.471	0.909	8.49	4.79	1363	461	7.09	20.03	16.13
84	1.796	600	720	811	682	87	3.24	3.77	24.0	44.6	18.41	.479	0.934	8.59	4.91	1288	436	7.54	21.30	17.15
85	2.064	604	700	782	679	79	3.72	4.33	22.1	40.0	16.95	.491	0.933	8.85	4.96	1486	508	6.94	19.61	15.79
86	1.750	605	730	824	690	89	3.16	3.67	24.2	45.9	18.88	.472	0.925	8.36	4.84	1238	423	7.66	21.64	17.43
87	1.645	605	747	845	695	94	2.97	3.46	25.7	48.6	19.75	.476	0.944	8.39	4.90	1159	395	8.18	23.11	18.61
88	1.487	603	762	871	697	101	2.68	3.13	26.0	53.7	21.29	.436	0.886	7.69	4.56	1047	354	8.28	23.39	18.84
89	1.413	601	780	895	700	107	2.55	2.97	27.6	57.1	22.33	.439	0.904	7.70	4.61	991	333	8.85	25.00	20.13
90	1.778	603	729	822	690	90	3.21	3.74	24.9	46.1	18.96	.481	0.943	8.60	4.93	1269	430	7.84	22.15	17.84
91	2.492	626	696	762	689	65	4.50	5.23	19.5	31.6	13.91	.547	1.00	9.81	5.32	1787	613	6.10	17.23	13.88
92	2.251	630	711	786	695	69	4.06	4.72	20.3	34.0	14.72	.531	0.988	9.45	5.21	1600	549	6.38	18.02	14.51
93	2.077	635	726	800	700	69	3.75	4.36	20.9	33.7	14.38	.556	1.05	9.79	5.51	1463	504	6.62	18.70	15.06
94	2.304	639	712	780	698	63	4.16	4.84	18.7	30.9	13.38	.539	1.00	9.55	5.29	1630	562	5.89	16.64	13.40
95	2.655	646	706	764	699	56	4.79	5.57	17.7	26.4	11.85	.598	1.07	10.58	5.71	1878	653	5.58	15.76	12.69
96	2.429	649	720	785	706	62	4.38	5.10	19.1	29.8	13.10	.573	1.05	10.11	5.52	1711	590	6.04	17.06	13.74
97	2.647	650	713	774	705	59	4.78	5.55	18.5	28.0	12.53	.590	1.06	10.40	5.60	1865	646	5.84	16.50	13.29
98	1.187	623	802	910	712	99	2.14	2.50	23.0	55.0	21.02	.383	0.807	6.63	4.08	822	278	7.44	21.02	16.93
99	1.928	625	714	786	690	68	3.48	4.05	19.1	34.4	14.52	.494	0.943	8.79	5.00	1370	472	6.01	16.98	13.67
100	2.132	630	701	766	689	62	3.85	4.47	16.8	31.6	13.59	.474	0.889	8.48	4.73	1522	524	5.30	14.97	12.06

2231

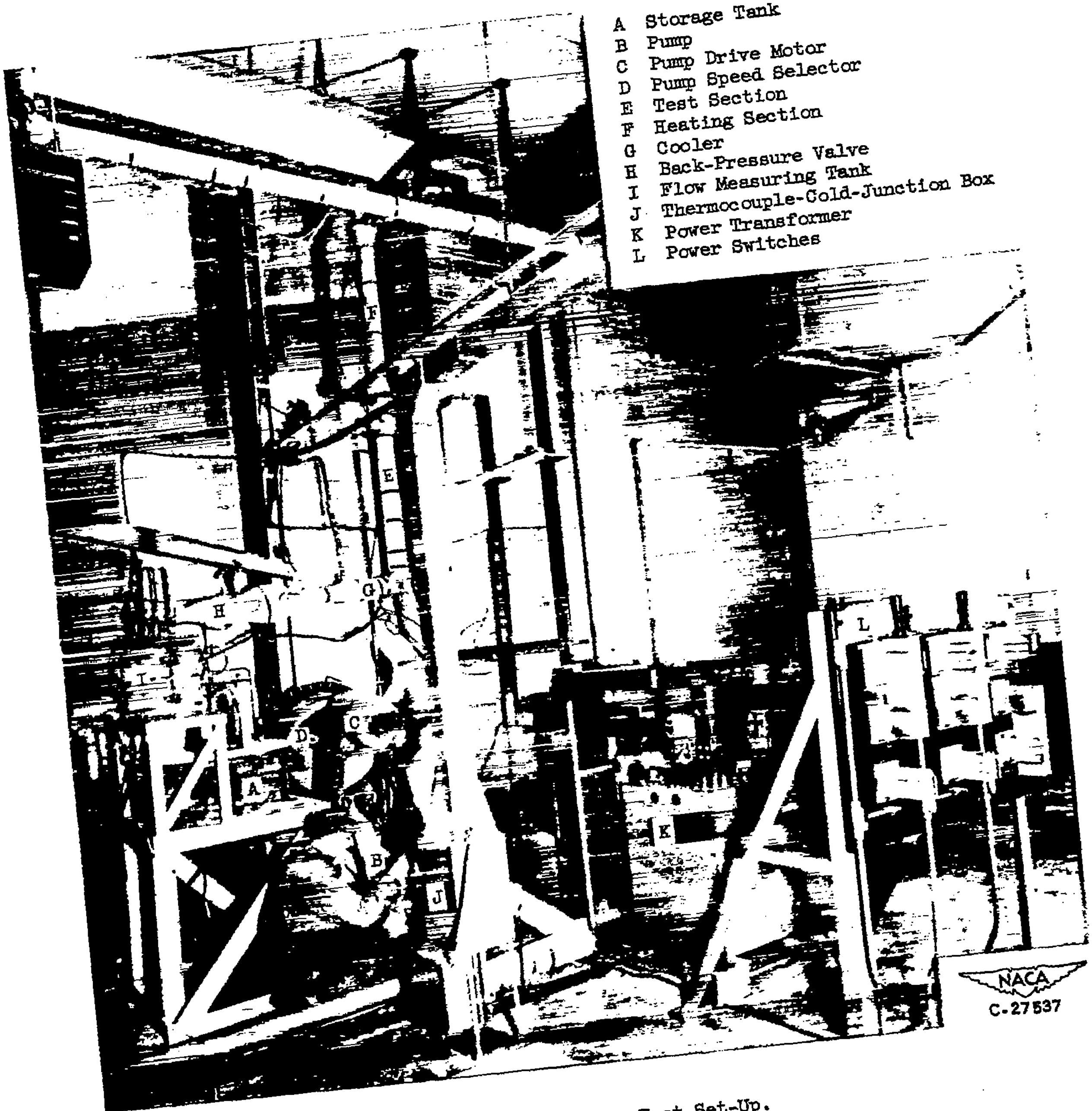


Figure 1. - Test Set-Up.

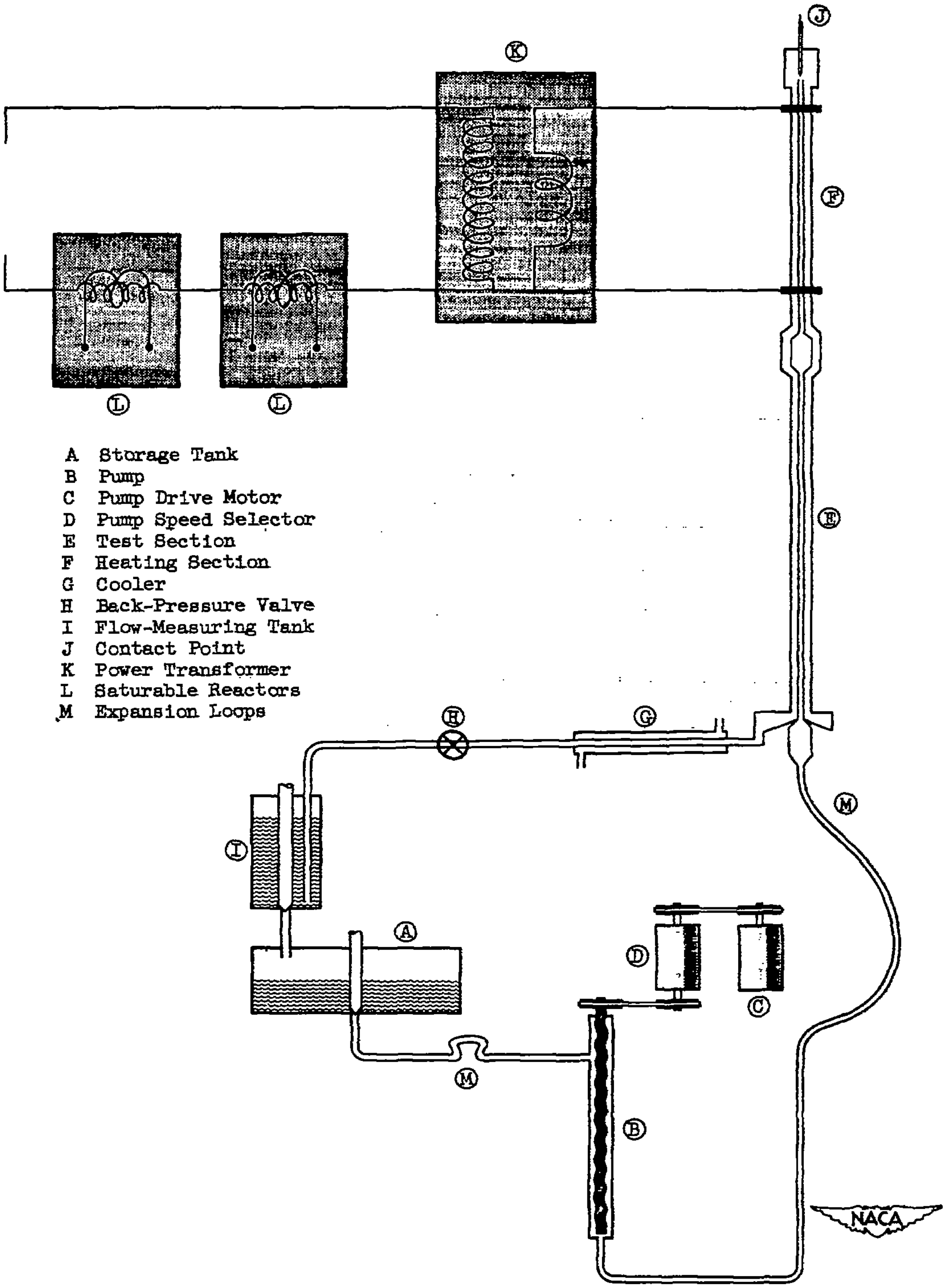
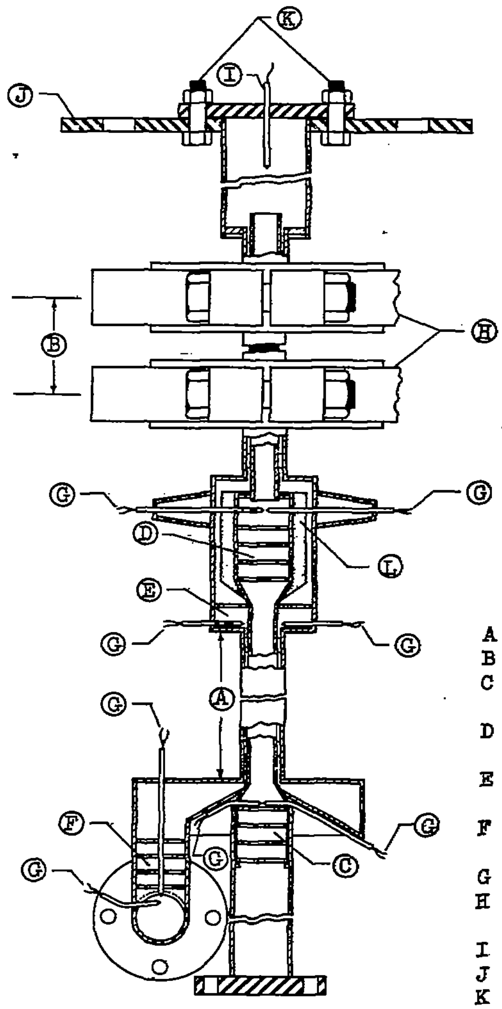


Figure 2. - Test Set-Up.



A Test Section
B Heating Section
C Center-Passage Inlet Mixing Section
D Center-Passage Outlet Mixing Section
E Annulus-Inlet Mixing Section
F Annulus-Outlet Mixing Section
G Thermocouples
H Main-Power Electrical Connections
I Contact Point
J Suspension Bracket
K Insulated Bolts
L Quartz-Wool Insulation

Figure 3. - Diagram of Test and Heating Sections.

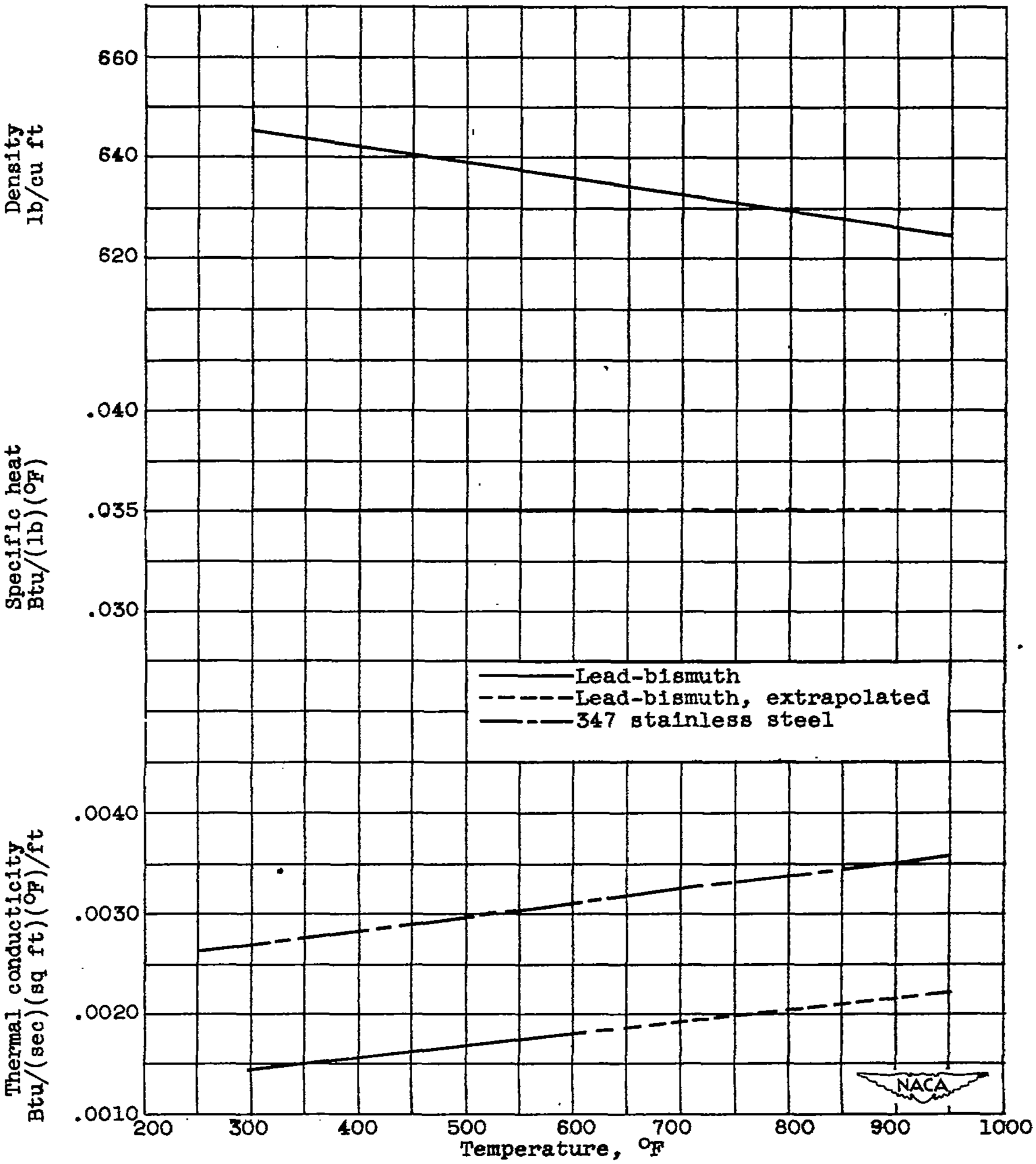
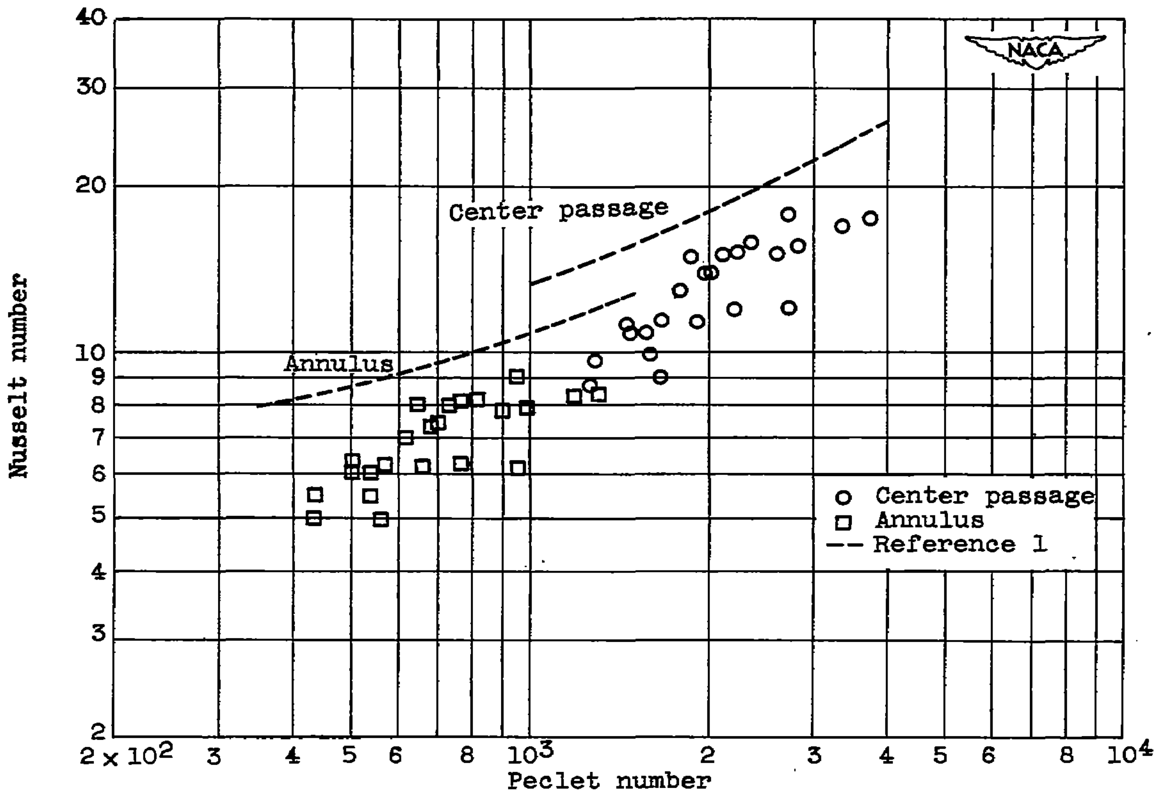
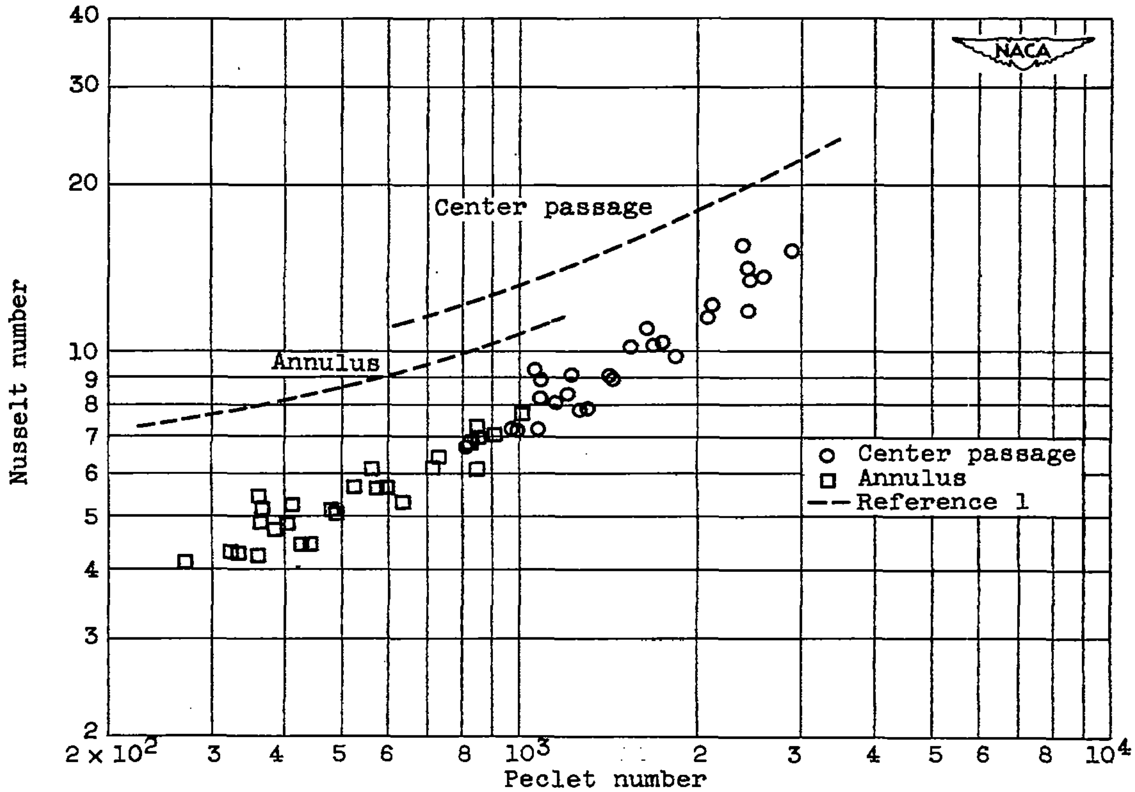


Figure 4. - Variation with temperature of some physical properties of lead-bismuth eutectic and 347 stainless steel.



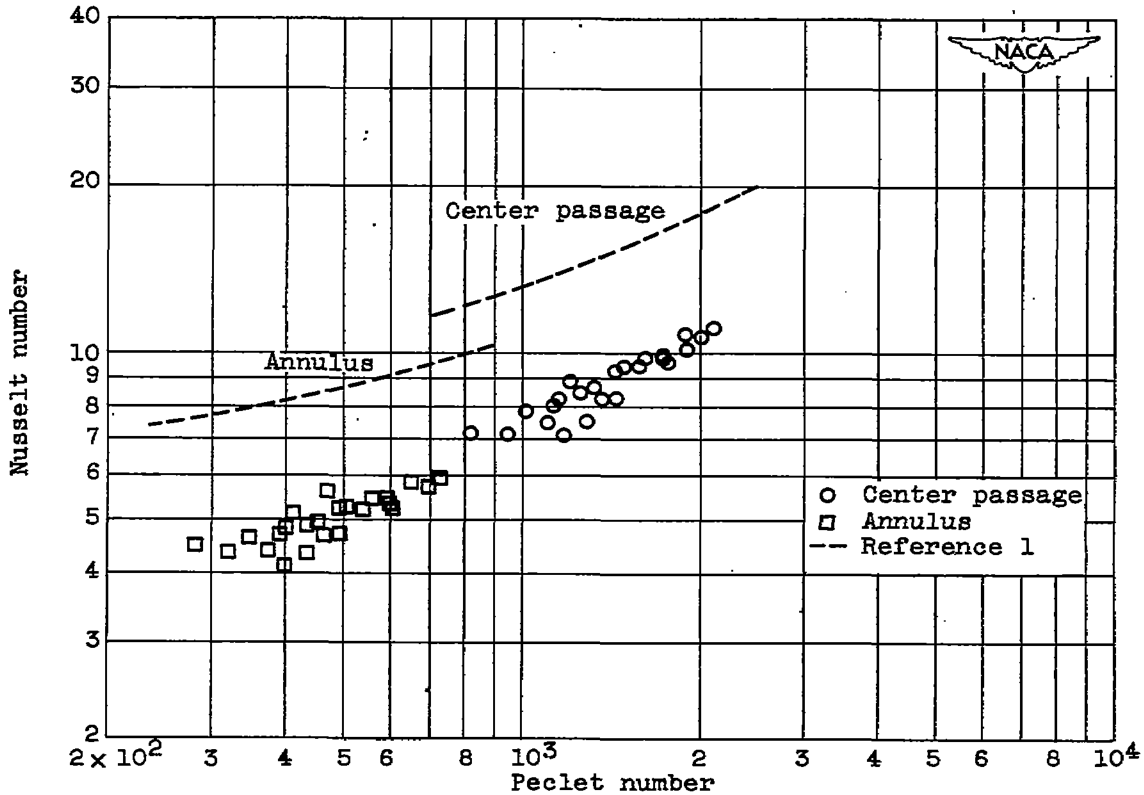
(a) Runs 1-23.

Figure 5. - Variation of Nusselt number with Peclet number for lead-bismuth eutectic with no wetting agent added.



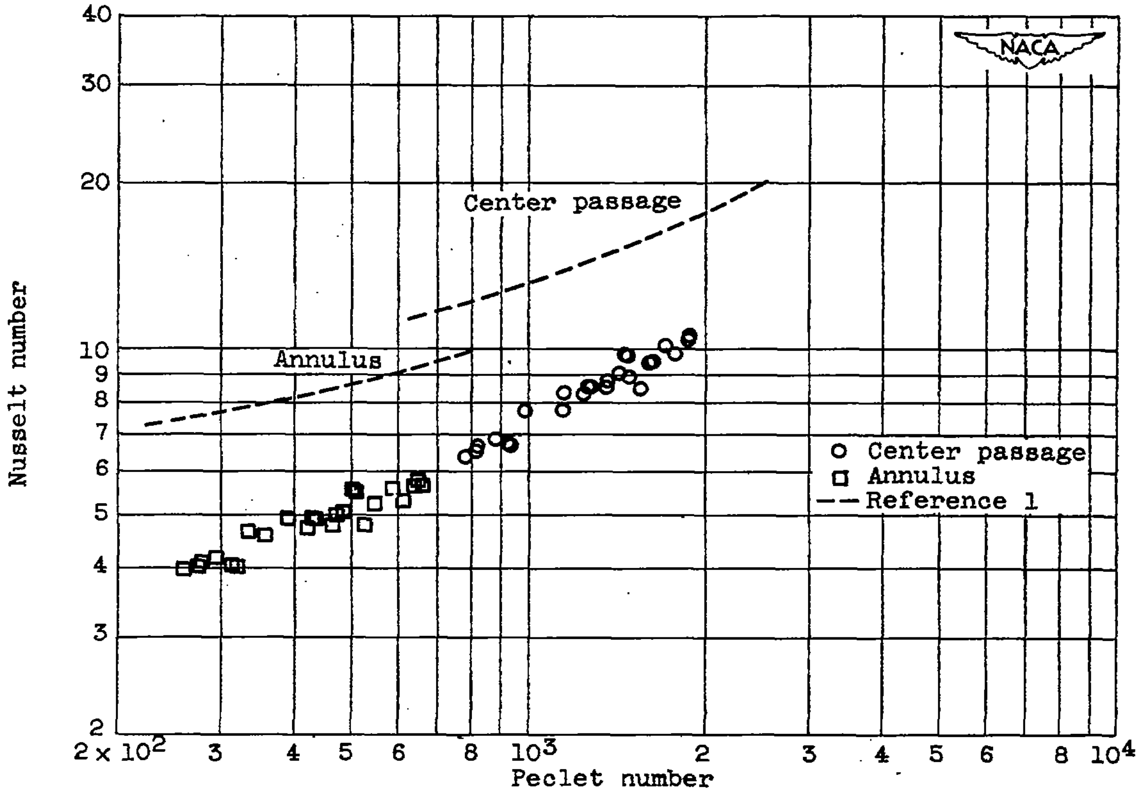
(b) Runs 24-50.

Figure 5. - Concluded. Variation of Nusselt number with Peclet number for lead-bismuth eutectic with no wetting agent added.



(a) Runs 51-75.

Figure 6. - Variation of Nusselt number with Peclet number for lead-bismuth eutectic with about 0.04 percent magnesium added as wetting agent.



(b) Runs 76-100.

Figure 6. - Concluded. Variation of Nusselt number with Peclet number for lead-bismuth eutectic with about 0.04 percent magnesium added as wetting agent.

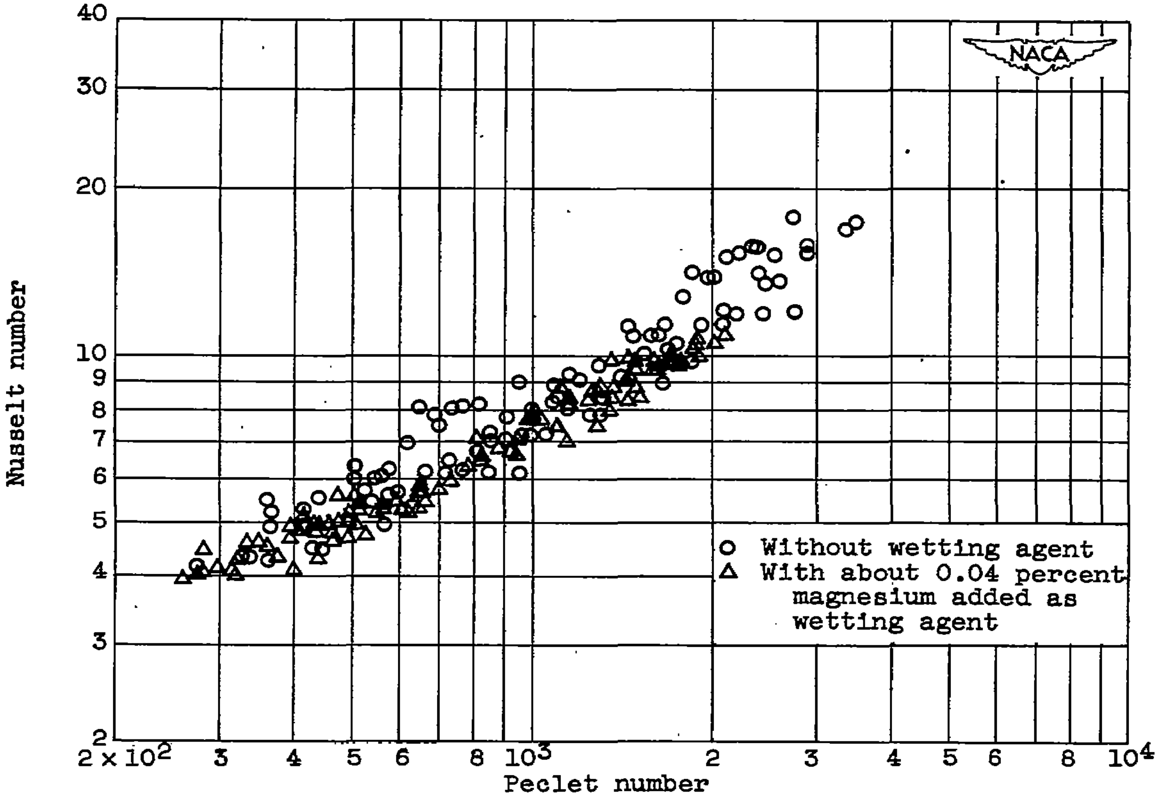
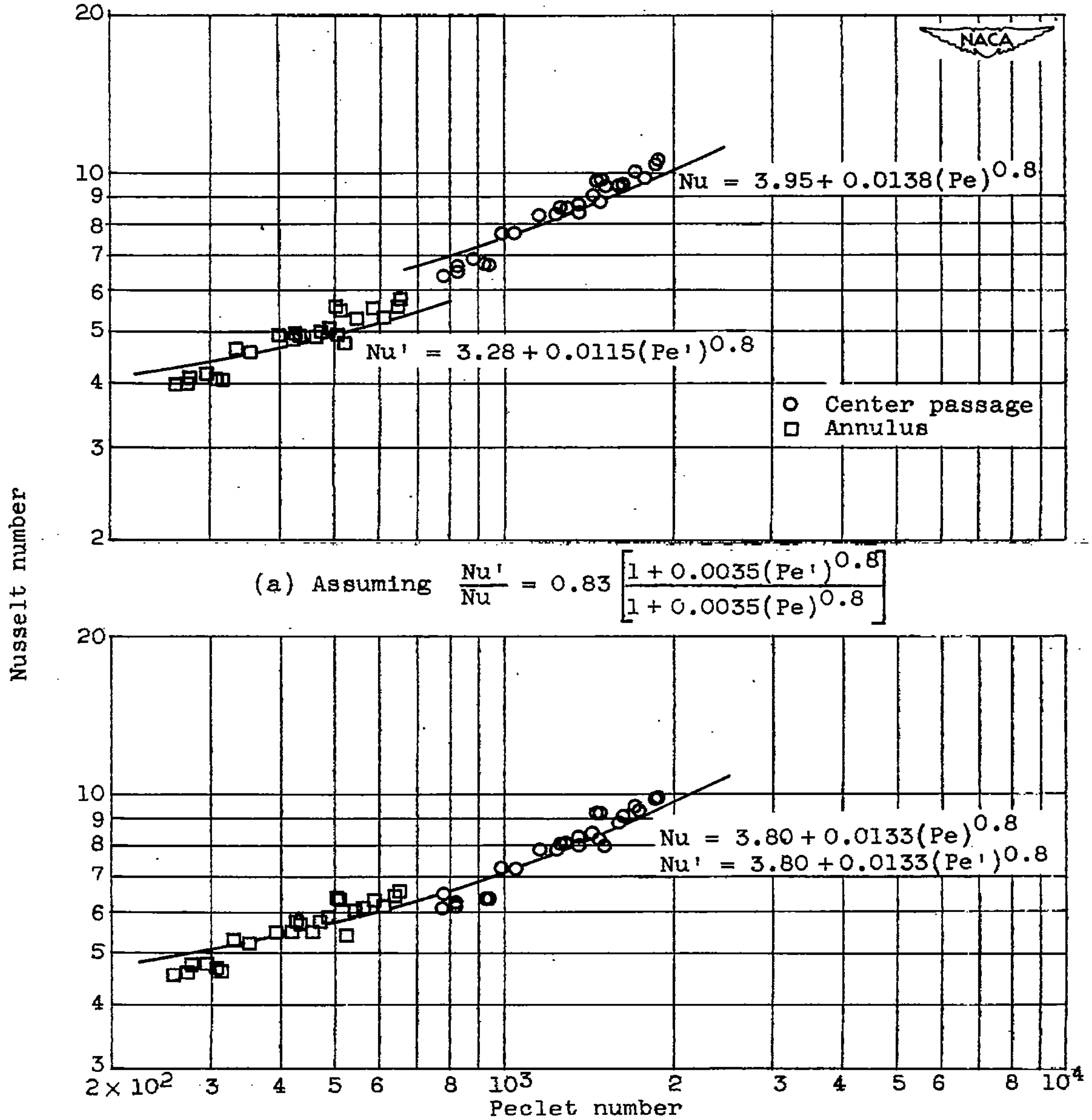


Figure 7. - Variation of Nusselt number with Peclet number for lead-bismuth eutectic, both without wetting agent, and with 0.040-0.045 percent magnesium added as wetting agent. Summary of all data points in figures 5 and 6.



(a) Assuming $\frac{Nu'}{Nu} = 0.83 \left[\frac{1 + 0.0035(Pe')^{0.8}}{1 + 0.0035(Pe)^{0.8}}\right]$

(b) Assuming $\frac{Nu'}{Nu} = 1.0 \left[\frac{1 + 0.0035(Pe')^{0.8}}{1 + 0.0035(Pe)^{0.8}}\right]$

Figure 8. - Effect of change in assumed equation for Nu'/Nu.

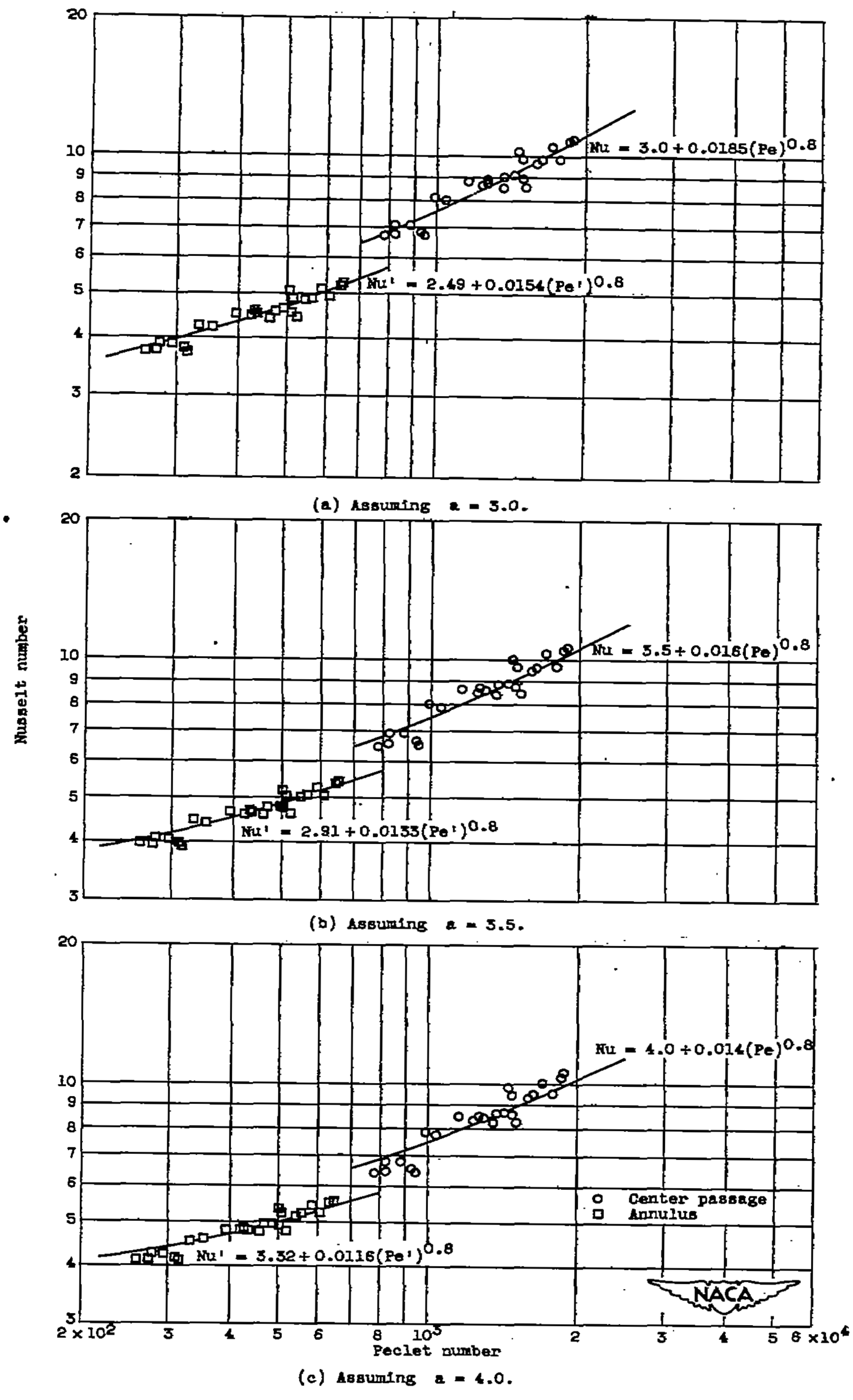


Figure 9. - Re-evaluation of data points of figure 6(b), using the assumptions $\frac{Nu'-a'}{Nu-a} = \frac{b'(Pe')^{0.8}}{b(Pe)^{0.8}}$, $\frac{a'}{a} = \frac{b'}{b} = 0.83$, and assuming various values for a.

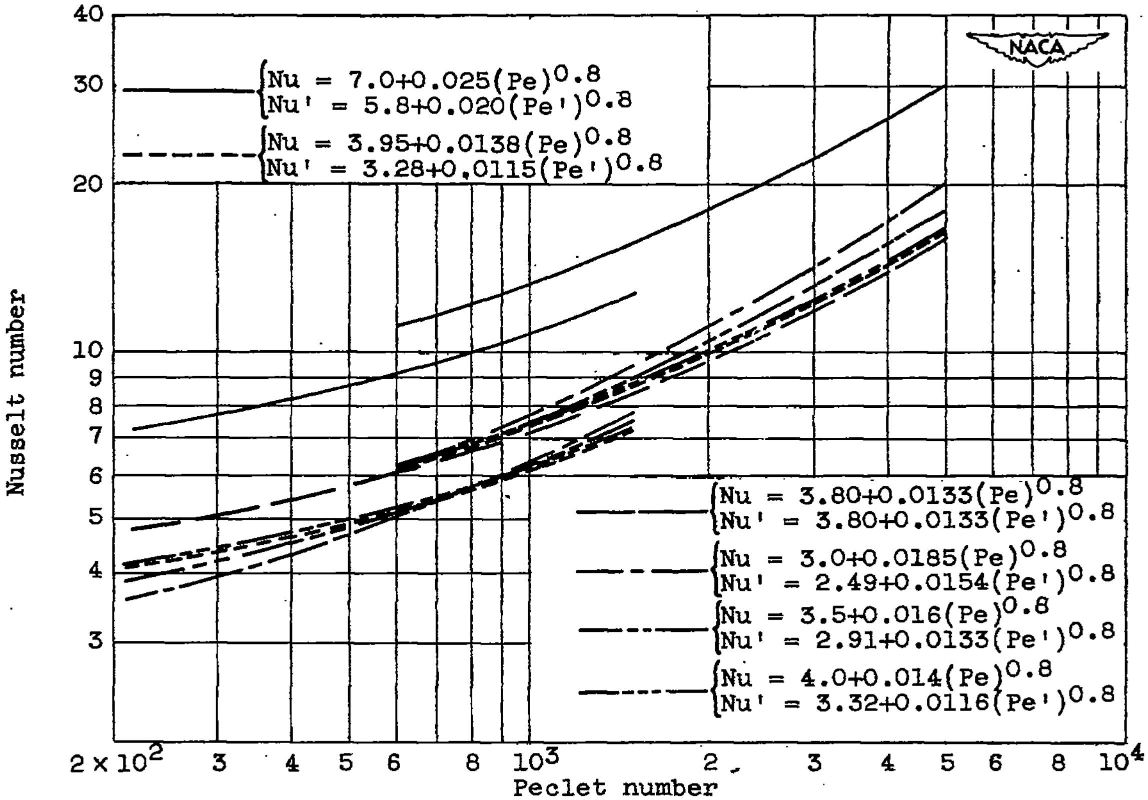


Figure 10. - Comparison of the various "correlating" equations.